SURVEY INTRODUCTION

INTRODUCTION

To understand and solve problems encountered in surveying, you must be familiar with certain concepts associated with geometry and trigonometry. These problems can be easily resolved by the use of simple trigonometric expressions. These formulas will enable us to quickly compute needed values without the necessity of having to measure them in the field.

ANGLES

This section will go over some of the simple geometry and trigonometric formulas and relationships that an Engineering Technician will use while in the field. First we will review the basic components of a triangle, which is composed of different types of angles.

An angle (<) is formed by two straight lines that intersect at a point, the vertex. The measurement of the angle will help determine the type of angle being formed. Angle measurements are expressed between 0-360 degrees, and they are subdivided by 60 minutes and 60 seconds. This measuring system is known as the sexigesimal system and is based on the number 60.

Angular Math

When adding or subtracting Degrees Minutes and Seconds, like terms must stay together. In other words, add (or subtract) the seconds first, followed by minutes then the degrees. Remember, 60"=1' and 60'=1°.

- **Example 1**:

$$\begin{array}{r} 35^\circ\ 15'\ 25'' \\ \underline{+\ 22^\circ\ 55'\ 48''} \\ 57^\circ\ 70'\ 73'' \end{array}$$

  - 73" and 70' have to be changed to the correct format:
    - There are only 60" in 1', so 73"-60" = 13".
    - Add the additional minute gained to 70'. 70'+1"=71',
    - then subtract 60' from 71' = 11'.
    - Now add 1° to 57°, and the final answer is: 58° 11' 13".

For subtraction,. the same rules apply, but you'll borrow multiples of 60.

Conversion Of Angles

It is sometimes desirable to change degrees, minutes, and seconds into a decimal equivalent. The following example shows how this may be done.

- **Example 2:** Change 16° 34' 22" into a decimal degree.

  - Step 1. Because there are 60 seconds in a minute, each second is 1/60 of a minute; therefore, 16° 34' 22" can be expressed as 16° 34.37' by dividing 60 into 22.

    16° 34' 22" = 16° 34' $\boxed{22'' \div 60}$ = 16° 34.37'

  - Step 2. There are also 60 minutes in a degree, so each minute is 1/60 of a degree; therefore, 16° 34.37' may be expressed as 16.5728, by dividing 60 into 34.37.

    16° 34' 22" = 16° $\boxed{34.37' \div 60}$ = 16.5728°

To change from decimal degree to a value expressed in degrees, minutes, and seconds, Multiply The Decimal Portion By 60. The following example shows how this may be done.

- **Example 3**: Change 16.5728° to degrees, minutes, and seconds.

  - Step 1. Multiply decimal portion by 60.

    .5728
    X 60
    34.3680

    The product is minutes; therefore, 16.5728° becomes 16° 34.368'.

  - Step 2. Multiply the decimal part of the minutes by 60 to find the seconds.

    .368
    X 60
    22.080 or 22 seconds

    Then, 16.5728° = 16° 34' 22".

Types of Angles

The angles a surveyor needs to be aware of are:

- An **acute angle** has a measurement of less than 90°.
- A **right angle** has a measurement of exactly 90°.
- An **obtuse angle** has a measurement that is more than 90° but less than 180°
- A **straight line** is an angle that is exactly 180°.

- A **complimentary angle** is two angles that add together to equal 90°.
- A **supplementary angle** is two angles that add together to equal 180°.

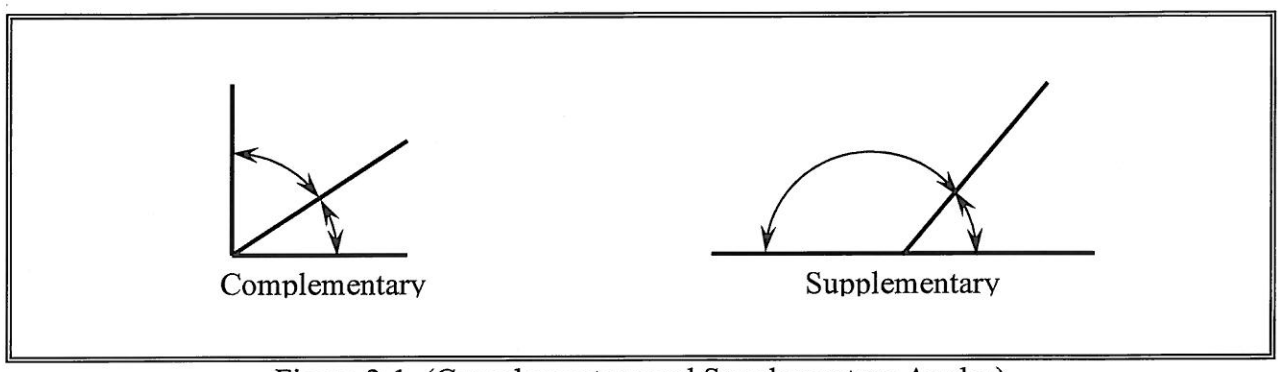


Figure 2-1 (Complementary and Supplementary Angles)

When two parallel lines AC and BD (Figure 2-2) are cut by a transverse line XY, the following relationship between angles exist:

- Opposite angles are equal, i.e., <2 = <3 , <1 = <4 , <5 = <8 , <6 = <7 .
- Alternate interior angles are equal. <3 = <6 , < 4= < 5 .
- Alternate exterior angles are equal. <2 = <7, < 1 = < 8.
- Angles on the same side of the transverse line are supplementary. <'s 1,3 and 5,7 and 2,4 and 6, 8 are all supplementary pairs.

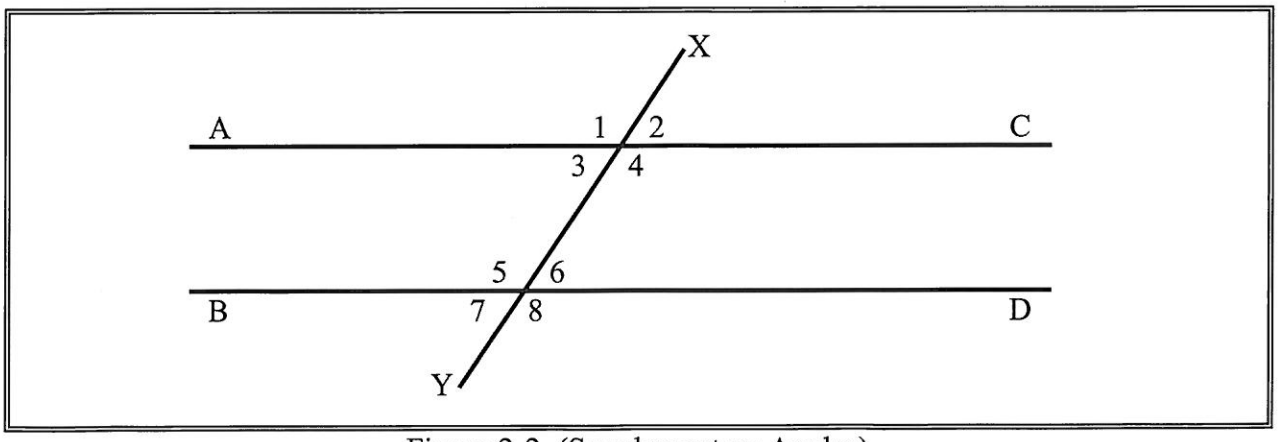


Figure 2-2 (Supplementary Angles)

POLYGONS

Polygons are many-sided figures, with sides that are line segments. Polygons are named according to the number of sides and angles they have. The most familiar polygons are the triangle, the rectangle, and the square.

Angles in Polygons. Sometimes it may be necessary to know the the different types of angles and the values of those angles associated within polygons. The three types of angles that exist are interior, exterior, and deflection.

- An **interior Angle** is formed inside a polygon at the intersection of any two sides.
- An **exterior Angle** is formed on the outside of a polygon at any vertex.
- A **deflection angle** is the angle between the extension of the preceding line and the present one.

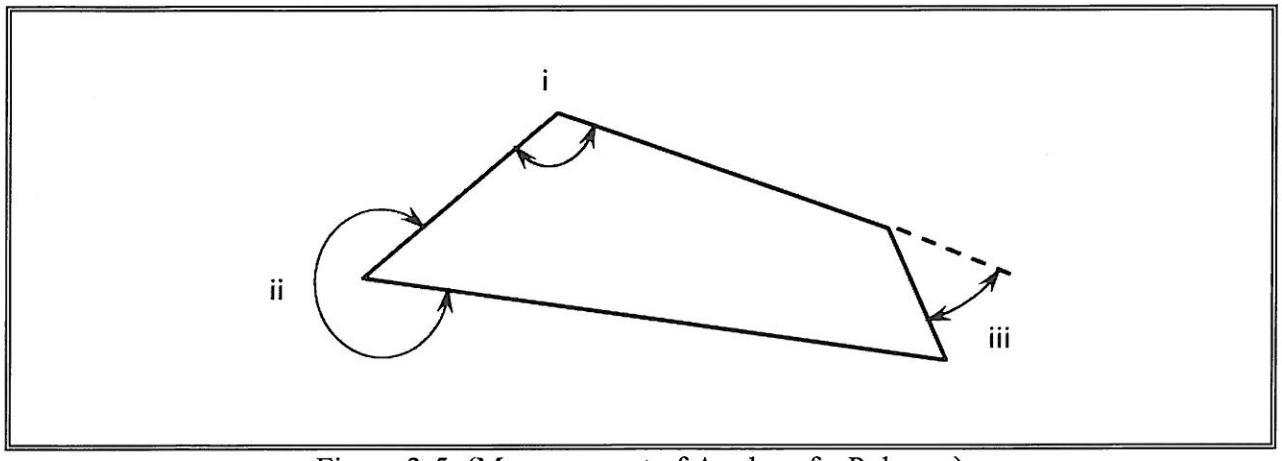


Figure 2-5 (Measurement of Angles of a Polygon)

Ways to determine measurements within a polygon:

- The sum of Interior angles = (n - 2) 180°
- The sum of Exterior angles = (n + 2) 180°
- The sum of Deflection angle = 360°
  - (n) is the number of sides in the polygon
  - The algebraic sum of the deflection angles turned to the right (+) and left (-) is equal to
  - 360

Triangle The sum of interior angles for all triangle is 180°. There are several types of triangles that may be used in trigonometric formulas.

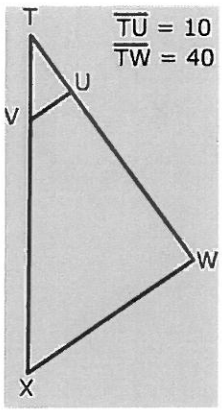
- An **equilateral triangle** has three equal sides and angles.
- An **isosceles triangle** has two equal sides and two opposite angles are also equal
- A **right triangle** has a an angle that equals 90°, or a right angle.

Formula to determine the area of a triangle is **A = ½bh**

Similar Triangles are often used in surveys to measure inaccessible distances . The formulas that can be used to help determine these distances are the Similarity Ratio or the Law of Similar Trianlges. These calculations are performed when triangles have corresponding (matching) sides, the angles are congruent (equal), and the ratio of their corresponding sides are proportional.

- Two triangles are similar if:
  - 3 angles of 1 triangle are the same as 3 angles of the other
  - 3 pairs of corresponding sides are in the same ratio
  - An angle of 1 triangle is the same as the angle of the other triangle and the sides containing these angles are in the same ratio.

Example 1: $\Delta TUV \sim \Delta TWX$



Since we know that the two triangles are similar, all that we have to do in order to find the simlarity ratio is to match a pair of corresponding sides! And then to divide!

Step 1) Match corersonding sides

TU and TW are corresponding sides

Step 2) Divide the corresponding sides

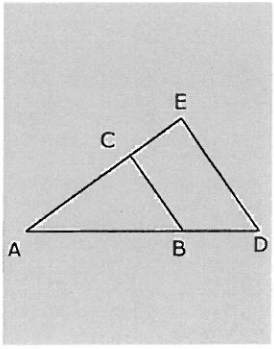
TW ÷ TU = ➡ 40/10 = 4/1

Therefore, the similarity ratio is 4/1 from ΔTWX to ΔTUV

Alternately, you could have found the similarity ratio from the smaller triangle, ΔTUV, to the larger ΔTWX. TU÷ TW = ➡ 10/40 = ¼

``

Example 2:



$\Delta ABC \sim \Delta ADE$

AB = 20

AD = 30

What is the similarity ratio?

The similarity ratio

Similarity Ratio = 2/3 if we divide AB by AD

AE = 33, how long is AC?

Side AC's Length 2/3 = AC/33

AC = 33 × 2/3 = 22

ED=27, how long is CB?

Side CB's Length CB = 27 × 2/3 = 18

Rectangles are polygons that have four sides and each angle is equal to 90°, and the opposing sides are equal in length. A common type of rectangle is a square which has four equal sides.

- The for any rectangle is **A = b (base) x h (height).**
- The area of a square can be expressed as $\mathbf{b^2}$.

Trapezoids are a four-sided polygon with 2 parallel sides. There is no definite relationship for the interior angles except that each pair of end angles are supplementary.

- The area of a trapezoid is: $\mathbf{A = \frac{(AC + BD)\ x\ h}{2}}$

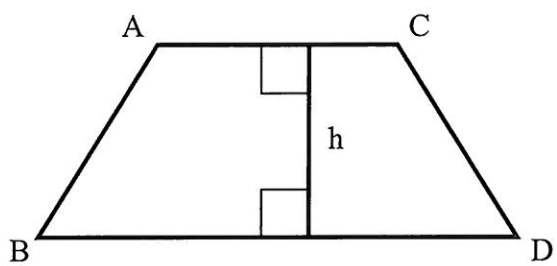


Figure 2-4 (Area of a Trapezoid)

SUMMARY

Geometric figures are made up of points connected by straight or curved lines with angles formed at these points. When these angles are 90 degrees they are called right angles, when they are less than 90^0 they are called acute, and when they are more than 90^0 they are called obtuse angles.

Angles are directional measurements. Degrees, minutes and seconds are commonly used as the unit of measure. The degree is 1/360 part of a complete circle. For certain mathematical operations it is necessary to convert angles of degrees, minutes and seconds in to degrees and decimals of degrees. Two angles whose sum is 90 degrees are called complementary angles and when their sum is 180 degrees they are called supplementary angles.

All enclosed geometric figures having three or more straight sides are called polygons. Each polygon will have a specific value for the sum of its angles. This value is dependent on the number of sides or angles of the figure.

VERTICAL CONTROL

Vertical control is usually carried out by direct leveling; however, indirect leveling may be used for a limited area or in rough terrain. One of the indirect leveling methods is trigonometric leveling, it is the most commonly used method while measuring the distance to an inaccessible object. The basic principle of trigonometric leveling is: You can apply the fundamentals of trigonometry to calculate the difference in elevation between the points when you know the vertical angle and either the horizontal or slope distance between two points, This method of indirect leveling is particularly adaptable to rough, uneven terrain where direct leveling methods are impracticable or too time consuming.

INACCESSIBLE OBJECTS

An inaccessible object is a point that needs to be measured but a surveyor is not able to occupy it because there is an an obstruction is in the way. Reasons for not being able to reach the base may include; terrain, manmade features, physical threats, or distance. If a surveyor runs into these types of obstacles they would be able to use **trigonometric leveling** to help determine the locations of the target.

Trigonometric Functions

To solve a triangle it is necessary to calculate the value of all unknown parts. There are six parts to every triangle; three sides and three angles. To solve a triangle using trigonometry it is necessary to have three known parts; one known must be the length of a side. Different functions or relationships are used to solve for different types of triangles and different parts of those triangles.

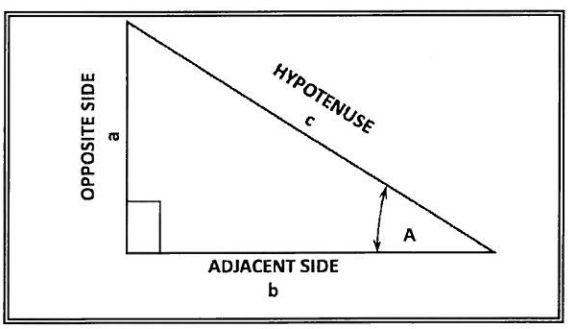


Figure 2-6 (Right Triangle)

Right Triangles. Trigonometric functions utilize sides and angles to help solve an unknown part of a right triangle. The length of one side divided by the length of another. Using any two of the three sides at a time in any order gives six different possibilities or functions. Each of these ratios or functions has a name and is defined as follows:

- **Sine of an angle** $= \dfrac{\text{opposite side}}{\text{hypotenuse}}$ $\quad \sin A = \dfrac{a}{c}$
- **Cosine of an angle** $= \dfrac{\text{adjacent side}}{\text{hypotenuse}}$ $\quad \cos A = \dfrac{b}{c}$
- **Tangent of an angle** $= \dfrac{\text{opposite side}}{\text{adjacent side}}$ $\quad \tan A = \dfrac{a}{b}$

Applying Trigonometric Functions

In order to apply trigonometric functions, it is first necessary to determine which function contains the values that you have to work with; and then, isolate the unknown value. The following are some examples.

Finding an Angle.

- **Example 1:**

1. Tan A $= \dfrac{Opposite}{Adjacent}$
2. Tan A $= \dfrac{a}{b}$
3. Tan A $= \dfrac{295}{300}$
4. Tan A $= 0.983333$
5. A $= 44° \; 31' \; 07$

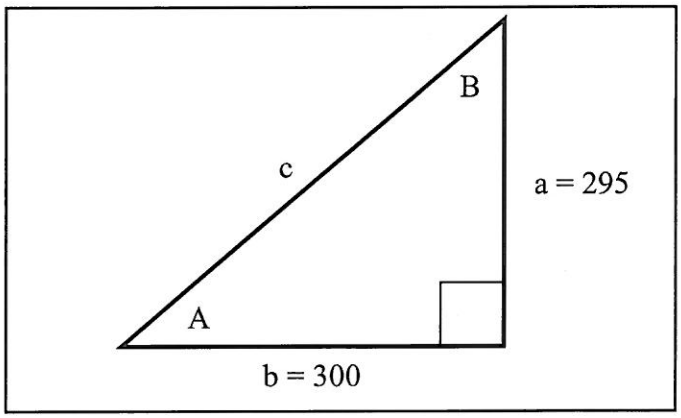


Figure 2-7 To Find an Angle

Finding a Side

- **Example 2:**

1. Cos A $= \dfrac{Adjacent}{Hypotenus}$
2. Cos A $= \dfrac{b}{c}$
3. c $= \dfrac{199}{0.65298}$
4. c $= 282.66$

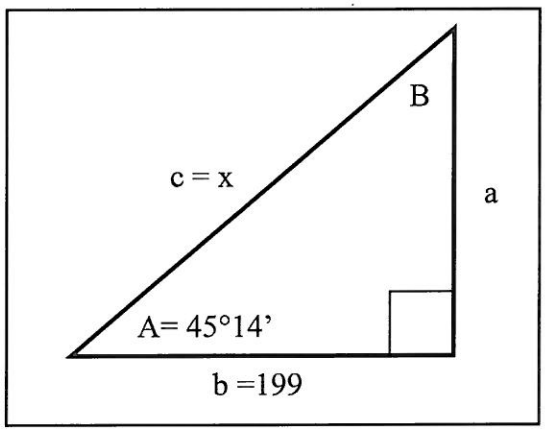


Figure 2 – 8 To Find a Side

The **Pythagorean Theorem** is used when two of the three sides are known and you need the third:

$$\boxed{c^2 = a^2 + b^2}$$

Oblique Triangles. Natural trigonometric functions for angles between 90° and 180° are determined by use of the following rules.

1. The sine of an obtuse angle is the sine of its supplementary angle and remains positive (sine in the Second Quadrant is positive).
2. The cosine of an obtuse angle is the cosine of its supplementary and is negative (cosine in the Second Quadrant is negative).

Laws and Rules for Solving Oblique Triangles. There are two trigonometric laws that are used to solve oblique triangles. They are the Law of Sines and Law of Cosines and they important equations because of their versatility and relatively simple usage and maneuverability.

Law of Sines. This law states simply that the sides of any triangle are proportional to the sines of the angles opposite them. Stated as a formula:

$$\boxed{\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}}$$

**** The law of Sines is used in the following cases.**

1. When any **two angles** and a **side** are **known.**
2. When any two sides and an **angle opposite one side** are **known.**

Law of Cosines. This law states that the square of a side is equal to the sum of the squares of the other two sides minus twice the product of these two sides times the cosine of the angle opposite that side. This law may be written for each of the sides in the following way.

ISOLATE A SIDE

$$a^2 = b^2 + c^2 - 2bc\,(\text{Cos } A)$$
$$b^2 = a^2 + c^2 - 2ac\,(\text{Cos } B)$$
$$c^2 = a^2 + b^2 - 2ab\,(\text{Cos } C)$$

OR

ISOLATE AN ANGLE

$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\text{Cos } C = \frac{a^2 + b^2 - c^2}{2ab}$$

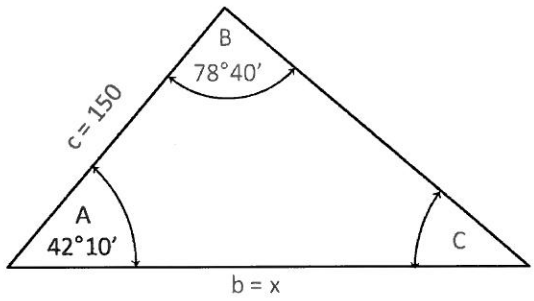
**** The Law of Cosines is used in the following cases:**

1. When **two sides** and the **included angle** are **known.**
2. When all three sides are **known.**

Applying Laws for Oblique Triangles: The following are examples of applying the laws of sines and cosines:

- **Example 1:** When given an oblique triangle, find side b.

  - We have two angles given, therefore, the third angle can be found by adding the two given angles and subtracting their sums from 180°. Then having the angle opposite the given side we can use the law of sines and isolate for side b.



STEPS:

1. First F=find Angle C
   a. C = 180° - (A + B)
   b. C = 180° - 120° 50'
   c. C = 59° 10'

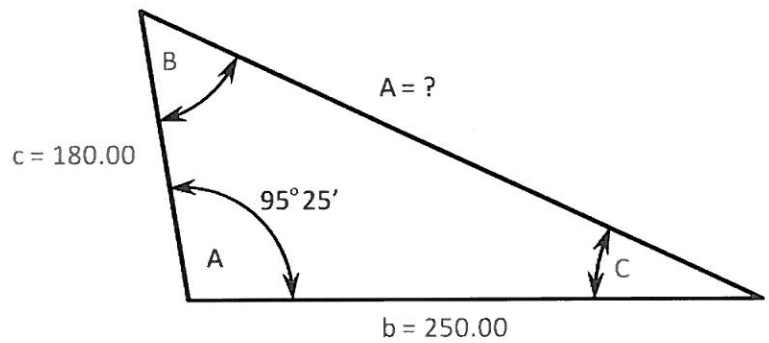
2. $$\frac{b}{Sin\ B} = \frac{c}{Sin\ C}$$

3. $$\frac{b}{Sin\ 78°40'} = \frac{150}{Sin\ 59°10'}$$

4. $$b = \frac{150(Sin\ 78°40')}{Sin\ 59°10'}$$

5. $$b = 171.28$$

Example 2: When given an oblique triangle, find side a.



STEPS:

1. $a^2 = b^2 + c^2 - 2bc \cos A$
2. $a^2 = 250.00^2 + 180.00^2 - 2\,(250.00)(180.00)(\cos 95°\ 25')$
3. $a^2 = 62500 + 32400 - 90000\ (-0.094398)$
4. $a^2 = 94900 + 8495.82$
5. $a^2 = 103395.82$
6. $a = 321.55$

SUMMARY

Unknown sides and angles of a right triangle can be calculated with the use of the three basic trigonometric functions. Two values must be known and one is a side.

Oblique triangles are triangles that have no angles of 90 degrees. The angles and sides of these triangles may be calculated when three values are known and one value is a side. To solve oblique triangles, the trigonometric laws of oblique triangles must be used. These laws are the law of sines and the law of cosines.

HORIZONTAL CONTROL

INTRODUCTION

A surveyor must be able to determine the Northings, Eastings and elevations of the location they are surveying. To do this they must establish horizontal control which means they must perform a traverse or locate a permanent bench mark to determine control points. Some control points are considered local because they are assumed and do not have actual coordinates assigned to them. These points can be tied into geodetic control once actual coordinates have been determined. These control points may help determine the location of detail points. Detail points are various points, objects, or details on the surface of the earth. Once the local control point has been given an acutal coordinate, the position of detail points should also be associated with the universal coordinates

One way to establish horizontal control is by performing a traverse. A traverse begins at at a point with an assumed or actual coordinate. First the surveyor determines an azimuth and then they are able to find angles and distances to help locate additional control points or detail points.

Horizontal Control is a fundamental network in which two or more points on the ground are precisely fixed in position by direction and distance.

Angular measurements

The following is a brief discussion on Angular measurement terms and definitions.

Azimuth By definition, an azimuth is the horizontal direction (angular measurement) of a line measured clockwise from North. The angular value of an azimuth is from 0 degrees to 360 degrees. See figure 5-1.

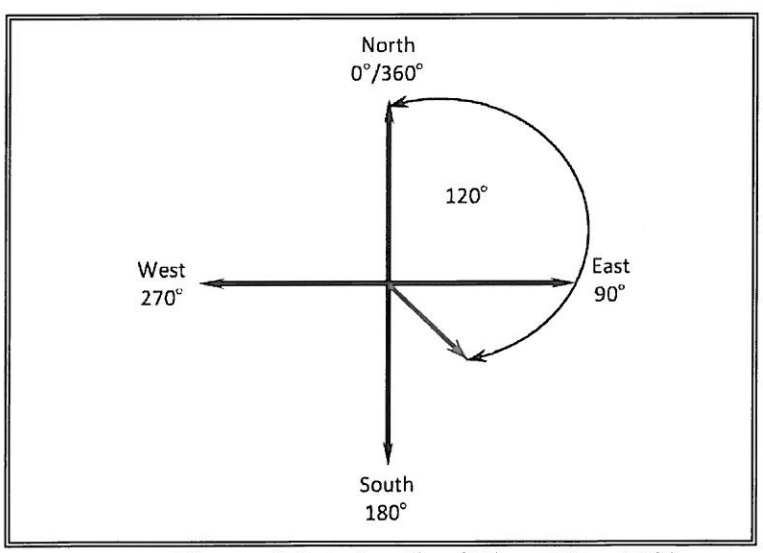


Figure 5-1 Azimuth of Line AB = 120°

Back azimuth The back azimuth is the horizontal distance going the opposite direction of the azimuth. To determine the back azimuth you will +/- 180° to the azimuth.

If the azimuth of a line lies between 0 and 180°, we add 180° to obtain the back azimuth. If the azimuth of a line is between 180° and 360°, we subtract 180° to obtain the back azimuth.

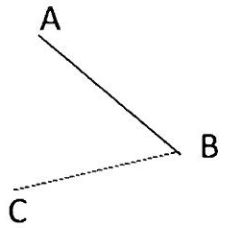
Azimuth Computations Of Lines Having A Common Vertex

In order to compute the azimuth (AZ) of an unknown line from another line, you must have the azimuth of the known line and the angle between the two lines.

If the measured angle between the two lines is turned to the **right** it is **added** to the azimuth of the known line to give you the azimuth of the unknown line.

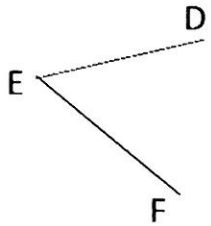
If the measured angle is turned to the **left**, it is **subtracted** from the azimuth of the known line to give you the azimuth of the unknown line.

Example 1:



Given Az BA = 300°
ABC = Left 61°
Unknown = Az BC
300° - 61° = 239°
Az BC = 239°1°

Example 2:



Given Az ED = 300°
DEF = Right 61°
Unknown = Az
300° + 61° = 1°
Az BC = 1°

Angular Error Of Closure Sum of station angles - [(N-2) x 180]

Allowable Angular Error (AAE). N x 20" or 1'√N (3rd order)
The allowable angular error is determined by the formula AE = 1' $\sqrt{n}$ or AE = 20"(n), where N is the number of traverse stations. **Since there are two formulas, you would choose the more strict of the two answers.** If azimuth error does not fall within allowable error, the surveyor must reobserve the station angles of the traverse in the field. or 20" per station.

Bearing

The bearing of a line is its direction within a quadrant that must never exceed 90°. These quadrants always start from the North or South axis and the angle must head in either an eastward or westward direction. To determine a bearing there are two necessary parts; there must be a bearing angle plus a directional quadrant. (Figure 5-4)

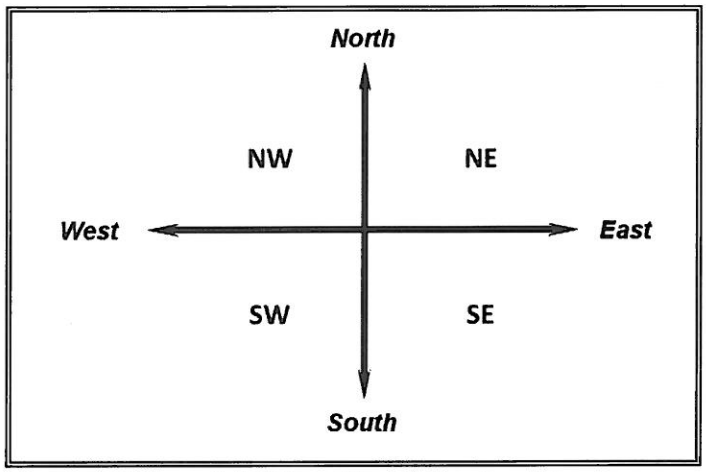


Figure 5-2 (Directional Quadrants)

To convert from an **Azimuth to a Bearing,** use the following rules:

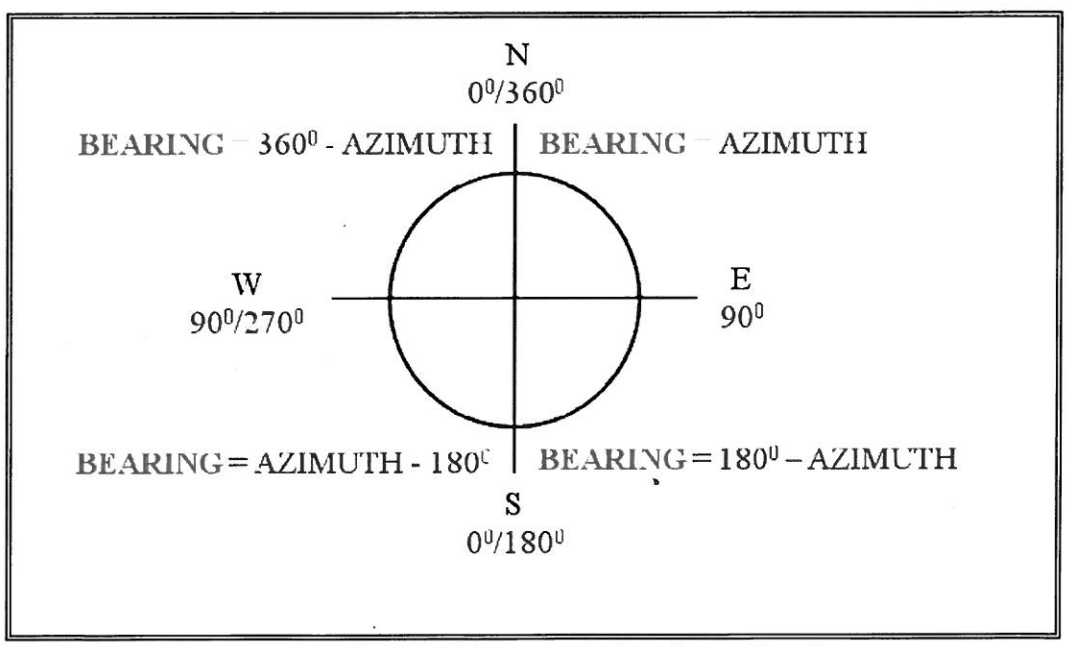


Figure 5-3 Convert From an Azimuth to a Bearing

To convert from a **Bearing to an Azimuth**, use the following rules:

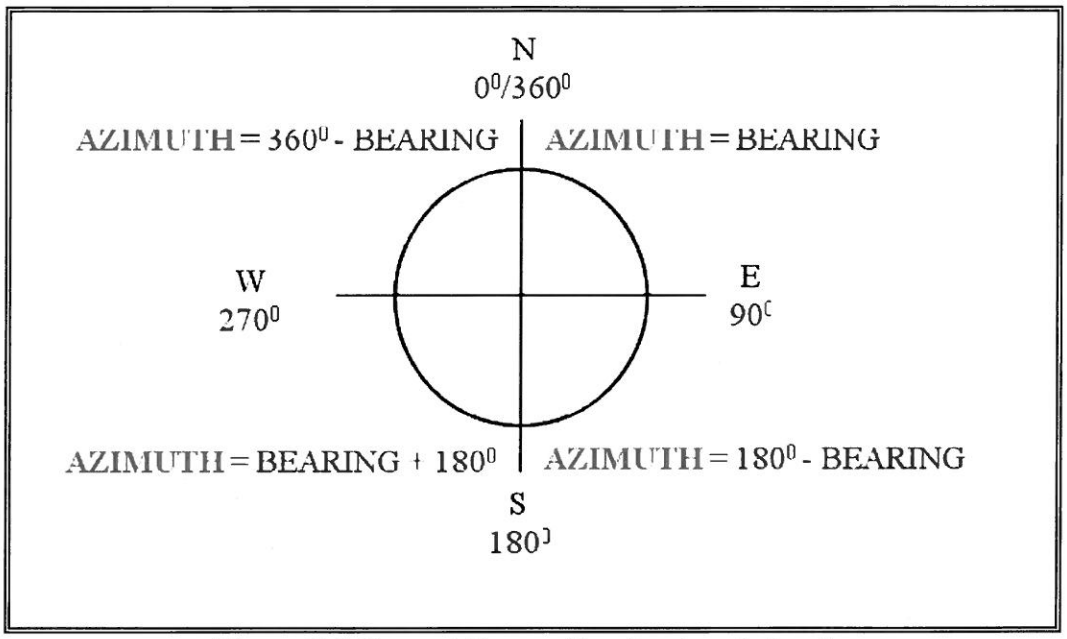


Figure 5-4 Convert From a Bearing to a Azimuth

Coordinate Computations

A surveyor can calculate an unknown point if they have the following information; the coordinates of a known point, the azimuth from a known point to an unknown point, and the distance between the two points.

In figure 7-5, the coordinate of station A is known and the coordinate of B is to be determined. The azimuth and distance from station A to B are determined by measuring the horizontal angle from the azimuth mark to station B and the distance from station A to B. The grid casting and northing lines through both stations are shown. Since the grid is a rectangular system and the casting and northing lines form right angles at the point of intersection, the computation of the difference in northing (side dN) and difference in casting (side dE) employs the formulas for the computation of a right triangle. The distance from A to B is the hypotenuse of the triangle, and the bearing angle (azimuth) is the known angle. The following formulas are used to compute dN and dE:

- dN = Cos Azimuth x Distance
- dE = Sin Azimuth x Distance

In figure 7-5, the traverse leg falls in the first (northeast) quadrant since the value of the casting increases as the line goes east, and the value of the northing increases as it goes north. Both the dE and dN are positive and are added to the casting and northing of station A to obtain the coordinate of station B.

If the surveyor uses a calculator with trigonometric functions to compute the traverse, the azimuth is entered directly and the machine provides the correct sign of the function and the dN and dE. If the functions are taken from tables, the computer provides the sign of the function by inspection. All lines going north have positive dNs; south lines have negative. Lines going east have positive dEs; west lines have negative. Figure 7-6 illustrates the relationship of quadrant to sign.

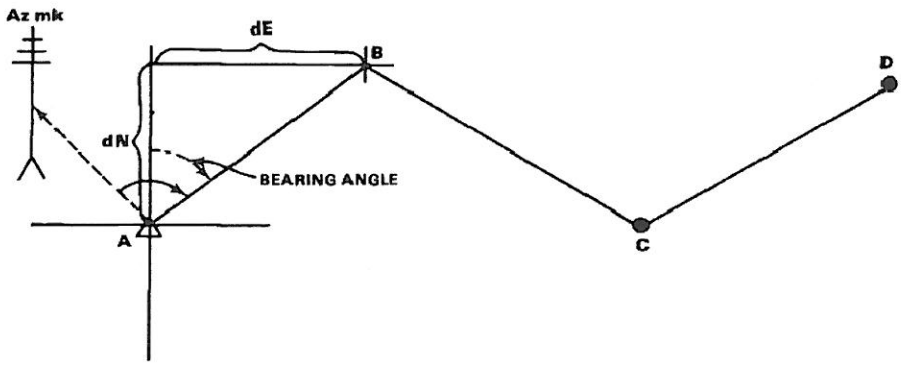


Figure 7-5. Requirements for dN and dE

In figure 7-5 the azimuth from A-B is 70°15'15" and the distance is 568.78.

- dN = Cos 70°15'15" X 568.78
- dN = +0.337848X 568.78= +192.16
- dE = Sin 70°15'15" x 568.78
- dE = +0.941200X 568.78 = +535.34

The azimuth from B-C is 161°12'30" and the distance is 548.74 (note SE quadrant).

- dN = Cos 161°12'30" X 548.74
- dN = -0.946696X 548.74= -519.49
- dE = Sin 161°12'30" x 548.74
- dE = +0.322128X 548.74 = +176.76

The azimuth from C-A is 294°40'45", and the distance is 783.74 (note NW quadrant).

- dN = Cos 294°40'45" X 783.74
- dN = 0.417537X 783.74 = +327.24
- dE = Sin 294°40'45" x 783.74
- dE = -0.908660X 783.74= -712.15

- **Example 1**: If given a coordinate and bearing with a distance, we can find the coordinates of the second point using the following formula:
  Lat (ΔN) = Cos Bearing x Distance
  Dep (ΔE) = Sin Bearing x Distance
  - Given: Sta A coordinates N 920.000 E 330.000
    Bearing to Sta B N 40°W
    Distance to Sta B 300'
  - Find: Coordinates of Sta B

  - **SOLUTION:**

ΔN = Cos Bearing x Distance
= (Cos 40°)(Distance)
= (.766044)(300')
= 229.81

ΔE = Sin Bearing x Distance
= (Sin 40°)(Distance)
= (.642788)(300')
= 192.836

229.813 ΔN (going north)
+ 920.00 Northing of Sta "A"

N 1149.813 Sta "B"

330.00 Easting of Sta "A"
- 192.836 ΔE (going West)

E 137.164 Sta "B"

- **Example 2**: If given two coordinates, we can find the bearing and distance between the two using the following formula: **$Dist^2 = Lat^2 + Dep^2$ ($Dist^2 = \Delta N^2 + \Delta E^2$)**
  - Given: Coord Sta 32 N 532.761 E 176.928
    Coord Sta 52 N 421.800 E 388.996
  - Find: Distance and Bearing from Sta 32 to Sta 52

- **SOLUTION:**

N 532.761 Coord Sta 32
- N 421.800 Coord Sta 52
ΔN (Lat) 110.961

E 176.928 Coord Sta 32
- E 388.996 Coord Sta 52
ΔE (Dep) - 212.068

$$Dist^2 = Lat^2 + Dep^2$$
$$Dist^2 = (110.9612)^2 + (212.0682)^2$$
$$Dist = 239.343$$

$$Tan = \frac{\Delta E}{\Delta N} = \frac{O}{A}$$
$$Tan = \frac{212.068}{110.961}$$

$$Tan = 1.911194$$
$$Tan = S\ 62°22'48''\ E$$

SURVEY FORMULAS

Polygonal Formulas

Sum of exterior angles: $(N+2) \times 180$
Sum of interior angles: $(N-2) \times 180$
Area of triangle: $\frac{1}{2}$ bh
Area of rectangle: $b \times h$
Area of trapezoid: $\frac{(b1 + b2) \times h}{2}$

Right Triangles

Sin of angle = o/h
Cos of angle = a/h
Tan of angle = o/a
Pythagorean theorem states: $c^2 = a^2 + b^2$

Oblique Triangles

Law of Sines states: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a = \frac{(b \sin A)}{\sin B}$; $b = \frac{(a \sin B)}{\sin A}$; $c = \frac{(b \sin C)}{\sin B}$

This is usually used when:
1. Any two angles and a side are known
2. Two sides and an angle opposite one of them is known

Law of cosines states:

$\cos A = (b^2 + c^2 - a^2 / 2bc$ $\qquad$ $a^2 = b^2 + c^2 - 2bc \cos A$
$\cos B = (a^2 + c^2 - b^2) / 2ac$ $\qquad$ OR $\qquad$ $b^2 = a^2 + c^2 - 2ac \cos B$
$\cos C = (a^2 + b^2 - c^2) / 2ab$ $\qquad$ $c^2 = a^2 + b^2 - 2ab \cos C$

This is usually used when:
1. Two sides and the included angle are known
2. All three sides are known

COORDINATE COMPUTATIONS

1. Determination of a coordinate from a known coordinate and a known bearing:

   Change in latitude (northing): Cos Bearing X Distance
   Change in departure (easting): Sin Bearing X Distance

   Take the answer and add/subtract the change to the known coordinate. Ensure you pay attention to the bearing to determine if it should be added or subtracted.

2. Determination of a bearing from two coordinates:

   First find difference in latitudes (northing) and difference in departures (easting)

   $dist^2 = lat^2 + dep^2$

   $$\text{tan of bearing} = \frac{\text{difference in dep}}{\text{difference in lat}}$$

   ΔN = Cos Adjusted Azimuth x Distance
   ΔE = Sin Adjusted Azimuth x Distance